٢٠١٠/٣/١٠ Fourier Transform سكشن ٥

8 Differentian

a) in time domain

$$g(t) \rightleftharpoons G(f)$$

n ــ عدد التفاضلات

$$\frac{d\, g^{(n)}(t)}{dt^{(n)}} \rightleftharpoons (J2\pi f)^{(n)} G(f)$$

I.L.T $\quad g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+J2\pi f t} \cdot df$

$$\frac{d\, g(t)}{dt} = (J2\pi f) \cdot \int_{-\infty}^{\infty} G(f) \cdot e^{+J2\pi f t}\, df$$

b) in frequency domain

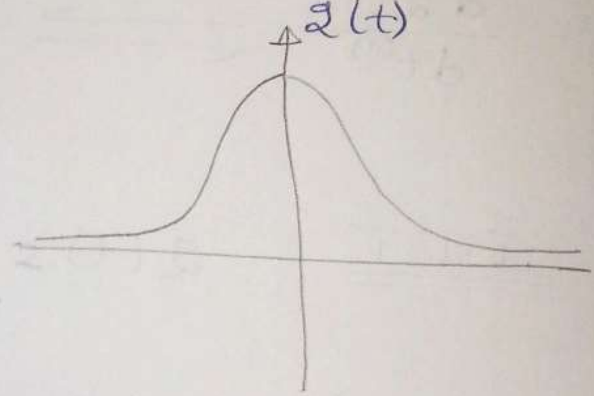
$$g(t) \rightleftharpoons G(f)$$

$$(-J2\pi t)^{n} \cdot g^{n}(t) \rightleftharpoons \frac{d\, G^{n}(f)}{df^{n}}$$

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-J2\pi f t} \cdot dt$$

$$\frac{dG(f)}{df} = (-J2\pi t) \int_{-\infty}^{\infty} g(t) \cdot e^{-J2\pi f t} \cdot dt$$

$\boxed{Ex}$ For the gaussian pulse shown find:-



a) area under curve $g(t)$.

b) " " " $G(f)$

c) F.T of $\frac{dg(t)}{dt}$

d) I.F.T of $\frac{dG(f)}{df}$

Hint:- $g(t) = e^{-\pi t^2}$

$G(f) = e^{-\pi f^2}$

a) $\text{Area} = \int_{-\infty}^{\infty} g(t) \cdot dt$

$\text{Area} = \int_{-\infty}^{\infty} g(t) \cdot e^{-J2\pi f t} \cdot dt \longrightarrow \text{put } f = 0$

Area under $g(t) = G(0)$ $\quad \because G(f) = e^{-\pi f^2}$

$G(0) = 1$

b) Area under $G(f)$ $\longrightarrow$ put $t = 0$

$= g(0) = 1$

c)

$$g(t) \rightleftharpoons G(f)$$

$$\frac{d}{dt} g(t) \rightleftharpoons (J2\pi f) \cdot G(f)$$

F.T of $\frac{d}{dt} g(t)$

$$= (J2\pi f) \cdot e^{-\pi f^2}$$

(d) I.F.T of $\frac{dG(f)}{df}$

$$g(t) \rightleftharpoons G(f)$$

$$(-J2\pi t) \cdot g(t) \rightleftharpoons \frac{d}{df} G(f)$$

$$\text{I.F.T of } \frac{dG(f)}{df} = (-J2\pi t) \cdot e^{-\pi t^2}$$

$\boxed{9}$ Integeration in time domain:-

if $\quad g(t) \rightleftharpoons G(f)$

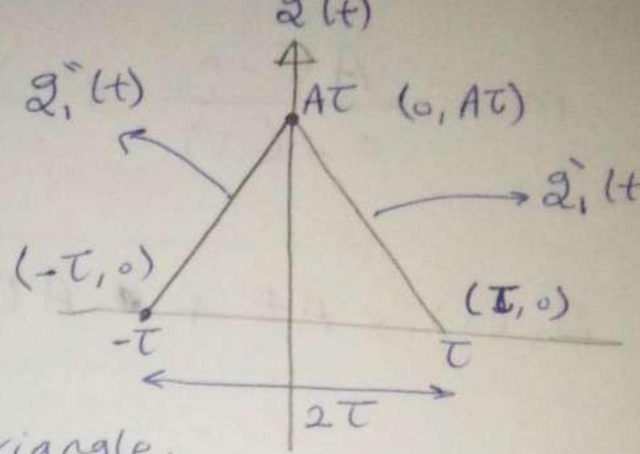
$$\int_{-\infty}^{\infty} g(t) \cdot dt \rightleftharpoons \frac{G(f)}{J2\pi f}$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+J2\pi f t} \cdot dt$$

$$\int_{-\infty}^{\infty} g(t) \cdot dt = \int_{-\infty}^{\infty} \frac{G(f)}{J2\pi f} \cdot e^{+J2\pi f t} \cdot dt$$

$\boxed{EX}$ Find F.T of $g_1(t)$ as shown

$g_1(t) = A\tau \; tri(t/\tau)$



$A\tau \rightarrow$ peak of triangle.

$t=0 \rightarrow$ center of triangle.

$\tau \rightarrow$ half of width of triangle.

← المثلث عبارة عن خطين فنوجد معادلة الخط المستقيم.

$$y = mx + c$$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$g_1'(t) = mt + c$$

$$m = \frac{A\tau - 0}{0 - \tau} = \frac{-A\tau}{\tau} = -A$$

$$\boxed{g_1'(t) = -At + A\tau} \longrightarrow (1)$$

$$g_1''(t) = mt + c$$

$$m = \frac{A\tau - 0}{0-(-\tau)} = \frac{A\tau}{\tau} = A$$

$$\boxed{g_1''(t) = At + A\tau} \longrightarrow (2)$$

$$g(t) \rightleftharpoons G(f)$$

$$\frac{dg(t)}{dt} \rightleftharpoons (j2\pi f)\, G(f)$$

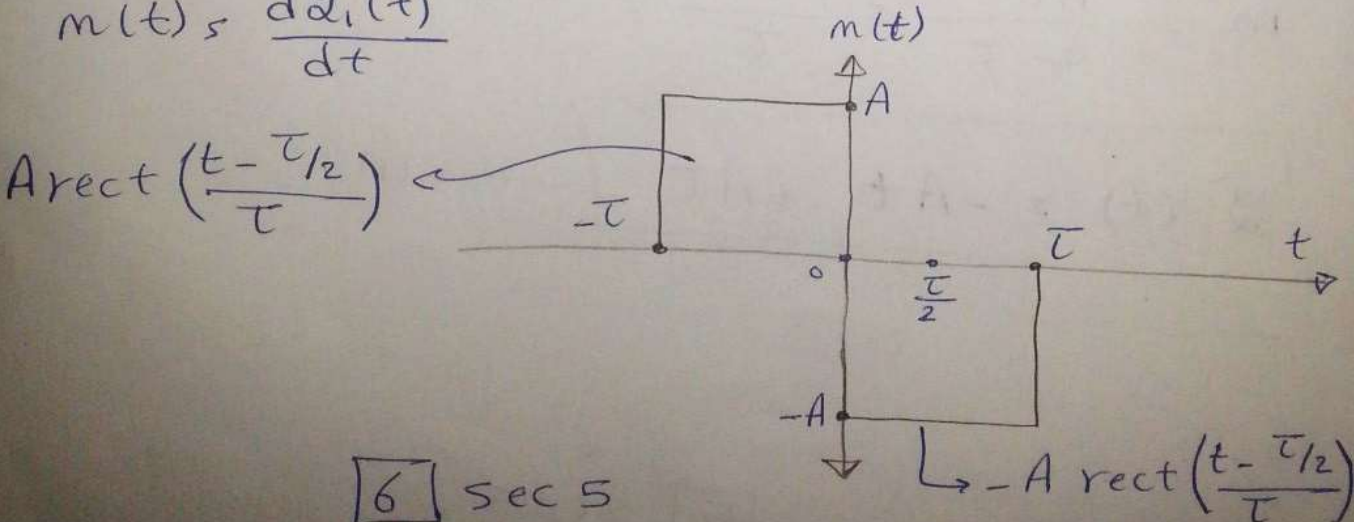
$$= m(t) \rightleftharpoons M(f) =$$

تفاضل $g(t)$ بالنسبة ل t هنسميه $m(t)$ ونجيب

$(F.T)$ ليها ونسميها $M(f)$ ويكون $M(f) = (j2\pi f)\, G(f)$

فحينها نعرف قيمة $G(f)$.

$$m(t) = \frac{dg_1(t)}{dt}$$



$$m(t) = A\,\text{rec}\left(\frac{t+\tau/2}{\tau}\right) - A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right)$$

$$M(f) = A\tau \cdot \text{sinc}(f\tau) \cdot e^{+J2\pi f \frac{\tau}{2}} - A\tau *$$

$$\text{sinc}(f\tau) \cdot e^{-J2\pi f \frac{\tau}{2}}$$

$$= A\tau \cdot \text{sinc}(f\tau)\left[e^{+J\pi f\tau} - e^{-J\pi f\tau}\right]$$

$$= (2J)\,A\tau\,\text{sinc}(f\tau) \cdot \sin(\pi f\tau)$$

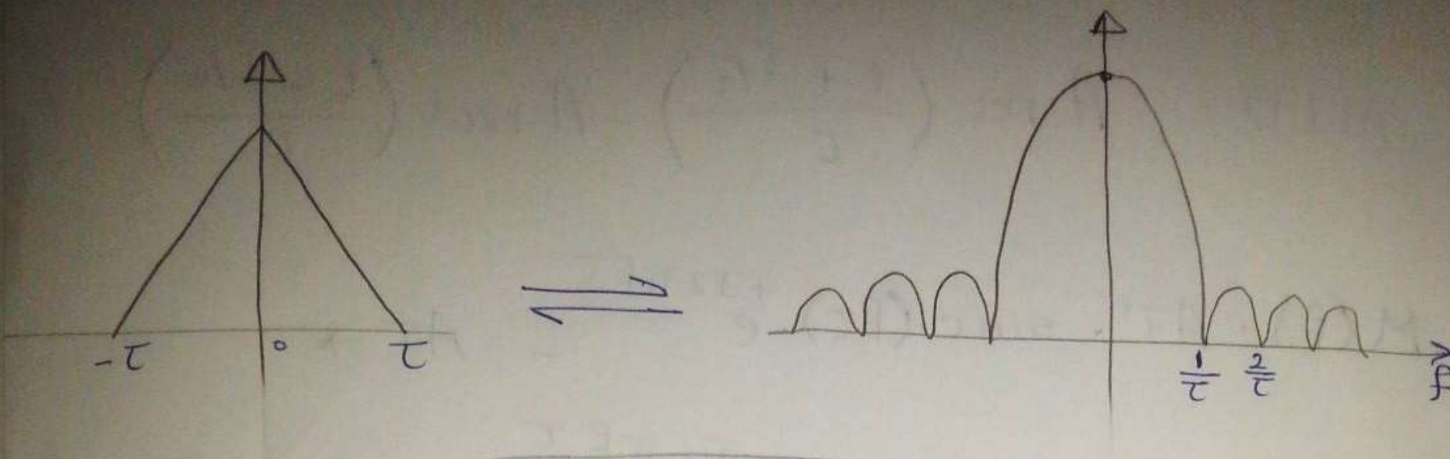
Note:- $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ $\quad x = f\tau$

$$(\pi f\tau) * \frac{\sin(\pi f\tau)}{\pi f\tau} = \text{sinc}(f\tau) * (\pi f\tau)$$

$$M(f) = (2J)(\pi f\tau)A\tau\ \text{sinc}^2(f\tau)$$

$$\boxed{G_1(f) = \frac{M(f)}{J2\pi f} = A\tau^2\ \text{sinc}^2(f\tau)}$$

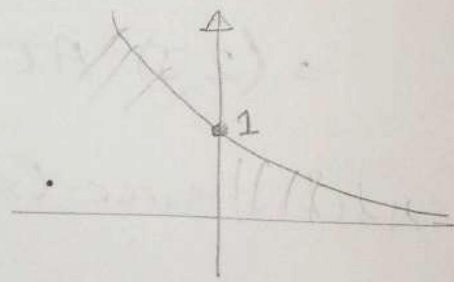
$$A\tau\ \text{tri}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau^2\ \text{sinc}^2(f\tau)$$



Delta Fn.

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\cdot \underbrace{e^{-j2\pi f t}}_{f(t)} \cdot dt$$

$$= f(0) = e^{0} = 1$$



$$\delta(t) \rightleftharpoons 1$$

$$1 \rightleftharpoons \delta(f)$$

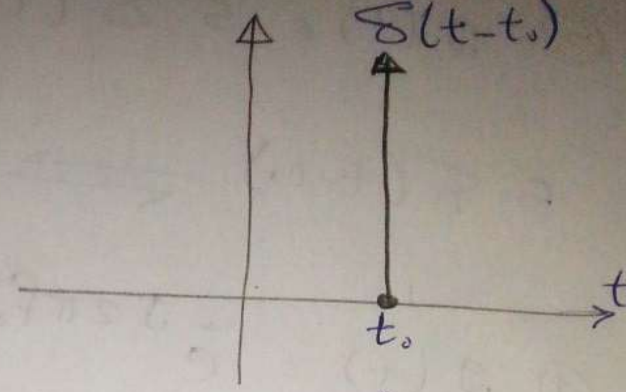
$\delta(t)$ ⇐ ~~at~~ $t=0$ تفرض مكان تعريف

ال delta .

$\delta(t - t_0)$

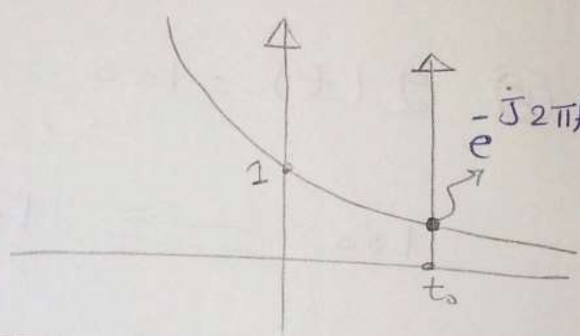
$\hookrightarrow t - t_0 = 0$

$t = t_0 \rightarrow$ center



$$F\left[\delta(t-t_0)\right] = \int_{-\infty}^{\infty} \delta(t-t_0) \cdot e^{-j2\pi f t} \cdot dt$$

$$= e^{-j2\pi f t_0}$$



Find F.T. of

① $A\delta(t)$

$$A\delta(t) \rightleftharpoons A$$

② $x(t) = 5\,\delta(t)$

$$5\,\delta(t) \rightleftharpoons 5$$

③ $g(t) = 5\,\delta(t-t_0)$

$$5\,\delta(t-t_0) \rightleftharpoons 5\,e^{-j2\pi f t_0}$$

④ $g(t) = e^{-j2\pi f_0 t}$

$$1 \cdot e^{-j2\pi f_0 t} \rightleftharpoons \delta(f+f_0)$$

⑤ $g(t) = 100$

$$100 \rightleftharpoons 100\,\delta(f)$$

⑥ $g(t) = A\cos(2\pi f_c t)$ $\boxed{\text{مهمة جداً}}$

$$= \frac{A}{2}\left[1 \cdot e^{j2\pi f_c t} + 1 \cdot e^{-j2\pi f_c t}\right]$$

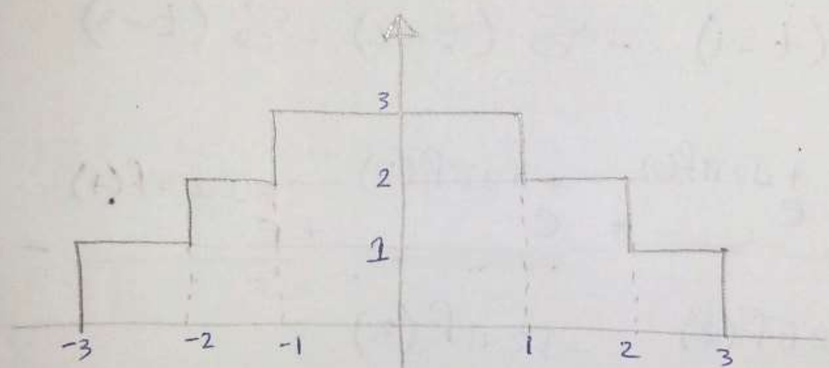
$$G(f) = \frac{A}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

(7) $g(t) = A \sin(2\pi f_c t)$

$$= \frac{A}{2J}\left[e^{J2\pi f_c t} - e^{-J2\pi f_c t}\right]$$

$$G(f) = \frac{A}{2J}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

$\boxed{Ex}$ Find F.T for $g(t)$ as shown



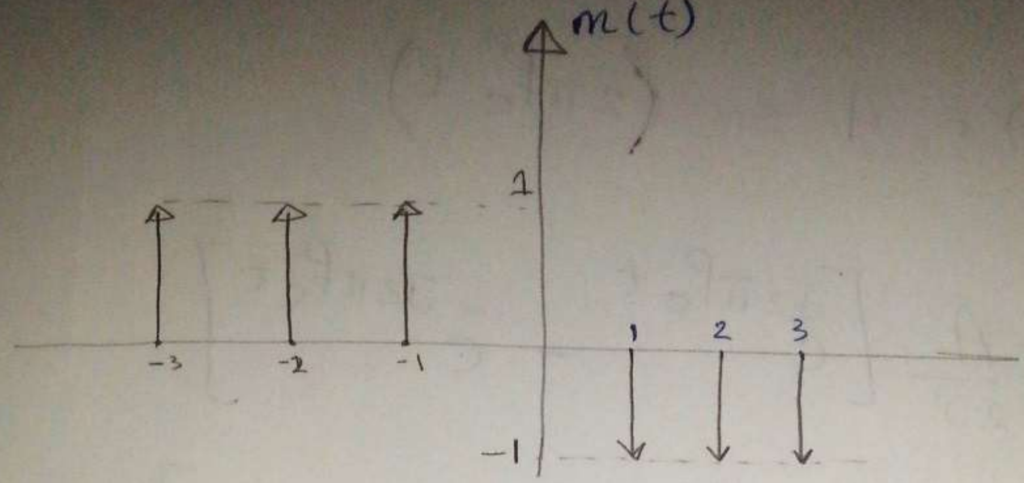
$$m(t) = \frac{d\,g(t)}{dt}$$

$$M(f) = (J2\pi f)\,G(f)$$

$g(t) \rightleftharpoons G(f)$

$m(t) = \frac{d\,g(t)}{dt} \rightleftharpoons (J2\pi f) * G(f)$

← هنفاضل الدالة من على الرسم ونعيد رسمها مرة أخرى.



← الأسهم دي تعبر عنه (transition) نعبر عنه ب $\delta(t)$

$$m(t) = 1 \cdot \delta(t+3) + \delta(t+2) + \delta(t+1)$$ ← الجزء الأيسر

$$- \delta(t-1) - \delta(t-2) - \delta(t-3)$$ ← الجزء الأيمن

$$M(f) = 1 \cdot e^{+J2\pi f(3)} + e^{+J2\pi f(2)} + e^{+J2\pi f(1)} - e^{-J2\pi f(1)}$$

$$- e^{-J2\pi f(2)} - e^{-J2\pi f(3)}$$

$$= 2J \sin(6\pi f) + 2J \sin(4\pi f) + 2J \sin(2\pi f)$$

$$G(f) = \frac{M(f)}{J2\pi f}$$

$$= \frac{1}{\pi f} \left[\sin(6\pi f) + \sin(4\pi f) + \sin(2\pi f) \right]$$